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Electroweak Symmetry Breaking and Extra Dimensions

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Abstract

Electroweak symmetry can be naturally broken by observed quark and gauge fields in various extra-dimensional configurations. No new *fundamental* fields are required below the quantum gravitational scale ($\sim 10 - 100$ TeV). We examine schemes in which the QCD gauge group alone, in compact extra dimensions, forms a composite Higgs doublet out of $(t,b)_L$ and a linear combination of the Kaluza-Klein modes of t_R. The effective theory at low energies is the Standard Model. The top-quark mass is controlled by the number of active t_R Kaluza-Klein modes below the string scale, and is in agreement with experiment.

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1 Electroweak asymmetry and extra dimensions

There are two major experimental observations which are not explainable solely in terms of the $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge interactions and the three generations of quarks and leptons: the electroweak symmetry breaking and the existence of gravity. It is now widely believed that a quantum theory of gravity necessitates a spacetime dimensionality greater than four. In this paper we show that the extra spatial dimensions, compactified at the $\sim$ TeV scale, also provide simple and natural mechanisms for electroweak symmetry breaking without the introduction of explicit Higgs fields.

We will argue that the Standard Model is the effective theory emerging, below the compactification scale, from a higher dimensional $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge theory with three generations of quarks and leptons and no fundamental Higgs field. A composite Higgs doublet arises naturally in the presence of certain strongly coupled four-quark operators. For concreteness, we will take these to involve typically the left-handed top-bottom doublet (ψ_L) and a vector-like quark [1]-[5], but we anticipate many possible variations of this particular arrangement. These particular four-quark operators *are always induced by QCD in compact dimensions*, via the exchange of the Kaluza-Klein (KK) excitations of the gluons [6]. Hence, the KK-gluons are effective "colorons" [7] and their effects can be quite large because the higher-dimensional QCD coupling constant increases above the compactification scale. The strength of these contact interactions depends on the ratio of the compactification scale, M_c, and the scale M_s of the underlying quantum gravitational effects. For M_c in the TeV range [8, 9, 10], M_s has to be around $10 - 100$ TeV such that the quantum gravitational effects cut-off the non-renormalizable higher-dimensional gauge interactions. Hence, the measured weakness of the gravitational interactions has to be explained by a modification of gravity at short-distance, for instance as proposed in refs. [11, 12, 13].

Indeed, the dependence of four-quark operator coefficients on the M_s/M_c ratio allows us to give a nice connection with string/M theory if one assumes that the gauge couplings unify at the string scale [14]. Due to the power-law running of the gauge couplings in extra dimensions [15], the value of the unified higher-dimensional coupling, $g_{4+\delta}(M_s)$, and the M_s/M_c ratio are determined almost exclusively by the number δ of compact dimensions accessible to the Standard Model gauge bosons. For $\delta \gtrsim 2$, $g_{4+\delta}(M_s)$ is of order one in M_s units, corresponding to a string coupling of order one. This is in accord with the argument based on dilaton stability [16] that string theory is in the truly strong-coupling regime.

Furthermore, the large value of $g_{4+\delta}$ implies that the strength of the four-quark operators induced by KK-gluon modes is non-perturbative, and may indeed bind a composite Higgs.

The only remaining ingredient for a viable theory of dynamical electroweak symmetry breaking is the above-mentioned vector-like quark. In four dimensions, a composite Higgs doublet may be bound out of the ψ_L and the right-handed top field, t_R [17, 18]. However, the Yukawa coupling of the Higgs doublet to its constituents is typically large, so that the top quark mass is too large (unless the theory is fine-tuned to nearly exact criticality, and the scale of the new interactions is taken to the GUT scale; alternatively, the measured top quark mass forces the VEV of this Higgs doublet to be smaller than the Standard Model Higgs VEV, $v/\sqrt{2}$ where $v \approx 246$ GeV is the electroweak scale).

On the other hand, if a new vector-like fermion is introduced with the same quantum numbers as t_R, it can then become the appropriate constituent of the Higgs boson together with ψ_L. The physical top mass is given in this case by a smaller eigenvalue of a mass matrix involving the vector-like and top quarks [1]. Therefore, such a seesaw mechanism neatly accomodates both the measured top quark mass and a Higgs VEV of $v/\sqrt{2}$.

It is quite striking that the Kaluza-Klein modes of the t_R have exactly the quantum numbers of this requisite vector-like quark. A key point of this paper is that the role of the vector-like quark can be naturally played by the tower of KK modes of the t_R. Therefore, compact extra dimensions appear to provide everything needed for a dynamical seesaw model of electroweak symmetry breaking[1]. Remarkably, however, while the vector-like excitations are required, the seesaw mechanism is no longer needed here, because the top Yukawa coupling is automatically suppressed by the (square-root of) number of active KK modes of the t_R with masses below M_s. Moreover, for typical ratios of M_s to the mass of the first quark KK excitation, the top Yukawa coupling computed to leading order in $1/N_c$ is between ~ 0.7 and ~ 1.4. Thus, the Standard Model value for the top Yukawa coupling (~ 1) is a natural consequence of our framework.

In Section 2 we first discuss chirality and anomaly cancellation in the case of one extra dimension. In Section 3 we present a detailed model of electroweak symmetry breaking valid below the quantum gravity scale which does not require any new field beyond the $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge fields and the three generations of fermions, in a higher dimensional configuration. We study the low energy effects of this model in Section 4. Finally, our conclusions are summarized in Section 5.

[1] Other studies of electroweak symmetry breaking in extra dimensions without a fundamental Higgs doublet can be found in [19].

2 Chirality and anomaly cancellation on a thick brane

In order to present the properties of the KK excitations of the t_R, we start with a general discussion of fermions in five-dimensions. The t_R may be the zero-mode of a five-dimensional fermion only if the gluons and hypercharge gauge boson propagate in the fifth dimension. Therefore the extra dimension has to be compact, with a radius below $\sim (3 \text{ TeV})^{-1}$ [9, 10, 14].

2.1 Chirality from orbifold projection

A constraint on the compactification of the extra dimension comes from the requirement that the t_R is a chiral, two-component fermion. The Lorentz group in five dimensions $SO(4,1)$ has only one spin-1/2 representation which turns out to be non-chiral. The fermions have four components, and the set of gamma matrices is formed of the usual four-dimensional ones, γ^μ, $\mu = 0, 1, 2, 3$, and of $i\gamma_5$. Therefore, a chiral zero-mode of a five-dimensional fermion may exist only if $SO(4,1)$ is broken. This can be done by compactifying the fifth dimension on an orbifold, or by imposing boundary conditions on the compact fifth dimension to distinguish the left- and right-handed components of the five-dimensional fermion.

Consider the four-dimensional Minkowski spacetime, with coordinates x^μ, and one additional transverse spatial dimension, with coordinate y. A simple way of breaking $SO(4,1)$ while preserving the four-dimensional Lorentz invariance and allowing chiral fermions is to compactify the fifth dimension, y, on an $S^1/\mathbf{Z}_2$ orbifold, *i.e.*, a circle of radius $R = L/\pi$ with the identification $y \to -y$. Five-dimensional fields are classified to be even or odd under $\mathbf{Z}_2$ parity. In terms of KK decomposition, the zero modes of the odd fields are projected out. The assignment of opposite $\mathbf{Z}_2$ parity to the left- and right-handed components of the five-dimensional fermion, $\chi_L(x,y) = -\chi_L(x,-y)$, $\chi_R(x,y) = \chi_R(x,-y)$, leaves massless only one four-dimensional right-handed chiral fermion.

Equivalently, one may start by considering a five-dimensional spacetime with boundaries along the fifth dimension at $y = 0$ and $y = L$. A four-component fermion field, $\chi(x,y)$, is defined on this space as a solution to the five-dimensional Dirac equation which obeys some conditions at $y = 0, L$. The simplest chiral boundary conditions,

$$\begin{aligned} P_L\chi(x,0) &= P_L\chi(x,L) = 0 \ , \\ \frac{\partial}{\partial y} P_R\chi(x,0) &= \frac{\partial}{\partial y} P_R\chi(x,L) = 0 \ , \end{aligned} \tag{2.1}$$

where $P_{L,R} = (1 \mp \gamma_5)/2$, lead to the quantization of momentum in the y direction, and give rise to the same KK decomposition as the $S^1/\mathbf{Z}_2$ orbifold projection discussed above.

These boundary conditions may result from the interactions between the bulk fields and the four-dimensional fields living on the branes located at $y = 0, L$. This in fact could be a physical explanation for the $S^1/\mathbf{Z}_2$ orbifold projection. In this paper, however, we do not attempt to derive a theory valid at any energy scale, but rather we study an effective field theory in a compact higher-dimensional spacetime defined below an ultra-violet cut-off M_s. We therefore impose only *four*-dimensional general covariance, and assume that the physics above M_s does not generate unwanted operators.

The complete set of orthogonal functions on the $[0, L]$ interval consistent with the boundary condition on χ_L (corresponding to odd fields under $y \to -y$) is given by

$$\sqrt{\frac{2}{L}} \sin\left(\frac{\pi j y}{L}\right) \ , \ j \geq 1 \ . \tag{2.2}$$

All these functions cancel on the boundaries, so that they indeed do not include a zero-mode on the compact interval, $[0, L]$. On the other hand, the boundary conditions for χ_R (corresponding to even fields) allow a complete set of orthogonal functions on $[0, L]$,

$$\sqrt{\frac{1}{L}} \, , \ \sqrt{\frac{2}{L}} \cos\left(\frac{\pi j y}{L}\right) \ , \ j \geq 1 \ , \tag{2.3}$$

which includes a zero-mode. The zero-mode of χ_R is identified as the right-handed top quark in the weak eigenstate basis, t_R. As a result, the decomposition of χ in KK modes is chiral:

$$\chi(x, y) = \frac{1}{\sqrt{L}} \left\{ t_R(x) + \sqrt{2} \sum_{j \geq 1} \left[P_R \chi_R^j(x) \cos\left(\frac{\pi j y}{L}\right) + P_L \chi_L^j(x) \sin\left(\frac{\pi j y}{L}\right) \right] \right\} \ . \tag{2.4}$$

A consequence of the boundary conditions (2.1) is that there is no fermion mass term in the five-dimensional Lagrangian. Nevertheless, the Dirac equation,

$$\left(\gamma^\mu \partial_\mu + i \gamma_5 \frac{\partial}{\partial y} \right) \chi(x, y) = 0 \ , \tag{2.5}$$

includes a γ_5 term so that it cannot be decomposed in separate equations for the left- and right-handed fermions. It is straightforward to derive the fermion propagator for this five-dimensional spacetime with the above boundary conditions:

$$\begin{aligned} \langle 0 | \, \chi(x', y') \, \overline{\chi}(x, y) \, | 0 \rangle \ &= \ \int \frac{d^4 k}{(2\pi)^4} e^{i k^\mu (x - x')_\mu} \frac{2}{L} \sum_{j \geq 0} \left[\cos\left(\frac{\pi j y'}{L}\right) P_R + \sin\left(\frac{\pi j y'}{L}\right) P_L \right] \\ &\times \frac{\gamma^\mu k_\mu + \gamma_5 \pi j / L}{k^\mu k_\mu - (\pi j / L)^2} \left[\sin\left(\frac{\pi j y}{L}\right) P_R + \cos\left(\frac{\pi j y}{L}\right) P_L \right] \frac{i}{1 + \delta_{j0}} \end{aligned} \tag{2.6}$$

We will use this propagator in section 3.2 to derive the Higgs potential.

2.2 Chiral Anomalies

Next we study what happens when the χ fermion transforms under some gauge symmetry. This is necessary in order to show that the model of electroweak symmetry breaking presented in the next section is anomaly-free.

Under the S^1/Z_2 orbifold projection discussed in the previous subsection, the ordinary four-dimensional spacetime components of the gauge fields A_μ must be even while the fifth components A_5 must be odd, so that they have consistent interactions with the χ fermion. Hence the fifth component A_5 has no zero mode, and its KK modes become the longitudinal components of the heavy A_μ modes. The zero-mode of A_5 may also be eliminated by imposing boundary conditions rather than an orbifold projection. If the gauge fields propagate in the fifth dimension only on the $0 \le y \le L$ interval, then the appropriate boundary conditions are given by $A_5(x,0) = A_5(x,L) = 0$ and $\partial A_\mu(x,0)/\partial y = \partial A_\mu(x,L)/\partial y = 0$. Although the graviton need not propagate at $y > L$ or $y < 0$, we refer loosely to the $[0, L]$ interval as a "thick brane" because $1/L$ is smaller than the string scale.

The five-dimensional Lorentz-invariant gauge theories have no chiral anomalies because the fermion representation is vector-like. However, the boundary conditions considered above prevent the existence of a χ_L zero-mode, which raises the question of anomalies. The $J^{a,r}_\chi \equiv \overline{\chi}\gamma^a T^r \chi$ current has an anomaly given by

$$D_a J^{a,r}_\chi = \frac{1}{24\pi^2 L}\epsilon^{\mu\nu\lambda\rho}\mathrm{Tr}\left[T^r \partial_\mu \left(A_\nu \partial_\lambda A_\rho + \frac{1}{2} A_\nu A_\lambda A_\rho\right)\right] , \tag{2.7}$$

where $A_\mu = -ig_5 A^{r'}_\mu T^{r'}$ is the gauge field, the trace is over the products of group generators T^r, D^a is the covariant derivative, and $\epsilon^{0123} = 1$. The index a runs from 0 to 4, with $\partial_4 \equiv \partial/\partial y$. Throughout this paper we use latin (greek) indices to denote the components of five (four) dimensional vectors.

Naively, one may think that this anomaly spoils the gauge invariance. It turns out, however, that the anomaly in this five-dimensional theory is more subtle. This is because the action may include a Chern-Simons term on the $[0, L]$ interval:

$$\mathcal{L}_{\mathrm{CS}}(A) = \frac{L-y}{96\pi^2 L}\epsilon^{abcde}\mathrm{Tr}\left[F_{ab}F_{cd}A_e - \left(F_{ab} - \frac{2}{5}A_a A_b\right) A_c A_d A_e\right] , \tag{2.8}$$

where F is the gauge field strength. In the presence of the Chern-Simons term, the gauge current becomes the sum of the fermion current and the Chern-Simons current. As a

result, the divergence of the total gauge current cancels everywhere on the *open* interval $(0, L)$:

$$D_a\left(J_\chi^{a,r} + J_{\rm CS}^{a,r}\right) = 0 \ . \tag{2.9}$$

Hence, the gauge theory with a Chern-Simons term is well defined (*i.e.*, non-anomalous) in the bulk of the fifth dimension. This is to be contrasted with the gauge anomaly in four-dimensions, which cannot be canceled by any counterterm in the action.

The physical interpretation of anomaly cancellation in the bulk of our five-dimensional theory is similar with that given in ref. [20] for the case of domain wall fermions in 2+1 dimensions. In the present case, the anomaly due to t_R on the $[0, L]$ interval produces gauge charges which are collected by the Chern-Simons current and transported towards the boundary. Therefore, in the bulk there is charge conservation. At the boundary, though, the charges are lost, so that the five dimensional theory with only one zero-mode fermion is indeed ill-behaved due to the anomaly. This can be seen by computing the variation of the action under a gauge transformation:

$$\delta \int d^4x \int_0^L dy \left(i\overline{\chi}\gamma^a D_a\chi + \mathcal{L}_{\rm CS}\right) = L \int d^4x\, D_a J_\chi^{a,r}\alpha^r\Big|_{y=0} , \tag{2.10}$$

where α^r is the gauge transformation parameter.

Therefore, there is need for other fermions such that the overall anomaly cancels, and the five-dimensional theory reduces to a non-anomalous four-dimensional gauge theory at scales below π/L. For simplicity we will assume that t_R is the only fermion with KK excitations below the string scale M_s. This is implemented in the effective field theory below M_s by localizing all the Standard Model fermions with the exception of t_R at certain positions in the fifth dimension. Evidently, the anomaly cancellation matches well in the effective theory below the compactification scale, where the only fermions present are the four-dimensional three generations of quarks and leptons.

The microscopic implication of anomaly cancellation in this case is that the charges which are driven by the Chern-Simons current (2.8) to the boundary are brought by another Chern-Simons current to the location of the other third generation fermions where they are absorbed by the corresponding four-dimensional anomalies. For example, a left-handed fermion located at $y = y_0$ and $z = 0$ requires a Chern-Simon term with a step function shape,

$$\frac{\theta(y-y_0)-1}{96\pi^2}\,\epsilon^{abcde}{\rm Tr}\left[F_{ab}F_{cd}A_e - \left(F_{ab} - \frac{2}{5}A_aA_b\right)A_cA_dA_e\right] , \tag{2.11}$$

to be added to $\mathcal{L}_{\rm CS}(A)$. As a result, the right-hand side of eq. (2.10) vanishes and the theory is gauge invariant.

We emphasize that the five-dimensional gauge theory is non-renormalizable. The gauge coupling has mass dimension $(-1/2)$, and it blows up at some scale $\sim M_s$. Therefore, any five-dimensional gauge theory should be seen only as an effective field theory which at the scale M_s is replaced by a more fundamental framework, such as string or M theory. The Chern-Simons terms discussed here are supposed to be produced within the theory that introduces the physical cut-off M_s.

Another possibility is that all third generation fermions are defined on the $[0, L]$ interval with chiral boundary conditions similar with those of χ. In this case the overall Chern-Simons current vanishes and the anomalies are canceled exactly as in the four-dimensional Standard Model. However, this would imply that all third generation fermions have KK excitations, which would complicate the analysis of the model presented in the next section. To keep the discussion simple, we will not investigate this possibility here.

3 A model: right-handed top and QCD in extra dimensions

In this Section we show that the dynamics in extra dimensions allows us to construct a model of dynamical electroweak symmetry breaking without the need for a fundamental Higgs field.

Consider a $(4+\delta)$-dimensional spacetime with the four-dimensional flat spacetime extended in the x^μ, $\mu = 0, ..., 3$ directions, and extra spatial dimensions with coordinates y and $z_1, ..., z_{\delta-1}$. Only some of the observed fields propagate in the extra dimensions. The simplest configuration is that where the gluons propagate in all these dimensions, the t_R is the zero mode of a fermion, χ, which is fixed at $z = 0$ but propagates on the $[0, L]$ interval in the y dimension, and the $\psi_L = (t, b)_L$ is fixed at $z = 0$ and $y = y_0$. We choose $\delta \gtrsim 3$ such that the effects of the gluons with momentum in the z dimensions are non-perturbative when the M_s scale is sufficiently large[2]. We will assume that the gluons propagate on intervals of size L and L_z in the y and $z_1, ..., z_{\delta-1}$ dimensions, respectively, with $L_z < L$. In Fig. 1 we sketch the extra-dimensional configuration.

As mentioned in the previous section, it is convenient to assume that all other quarks

[2]Note that in the case of a single compact dimension, the four-quark operators induced by the tree level exchange of an infinite tower of gluon KK modes are finite.

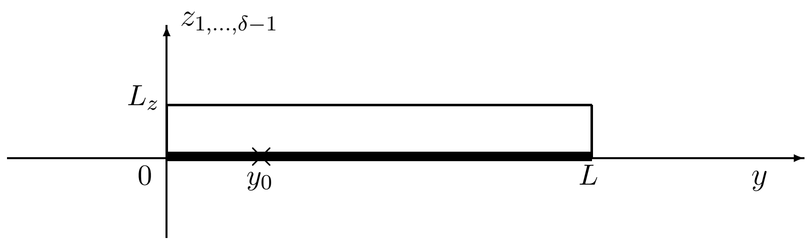

Figure 1: The profile of the compact space. The x-coordinates of the flat three-dimensional space are transverse to the plane of the page. The $z_1, ..., z_{\delta-1}$ coordinates are depicted collectively as one axis. The gluons propagate inside the rectangle, the χ propagates along the y axis, on the thick line, and the ψ_L is located at the point marked on the y axis.

and the leptons are localized on four-dimensional slices of $4 + \delta$-dimensional spacetime, so that we do not have to worry about their KK modes. Furthermore, if the left- and right-handed fermions (other than t_R and ψ_L) are split in the extra dimensions [21], then they cannot acquire large masses. Note that this splitting does not produce the kind of flavor-changing neutral currents discussed in [10] provided the light fermions of same chirality are located at the same places.

The $U(1)_Y$ gauge bosons have to propagate in the y dimension because χ carries hypercharge. The $SU(2)_W$ gauge bosons must propagate in the fifth dimension only if different weak-doublet fermions are localized at different places. Note that if gauge coupling unification is imposed, then it is preferable to have the $SU(2)_W \times U(1)_Y$ gauge bosons propagating in the same space as the gluons.

3.1 The five-dimensional theory

After compactifying and integrating over the z dimensions, we find a tower of KK modes of the gluons, which are grouped in levels of masses $\pi\sqrt{K}/L_z$ with K a positive integer, and degeneracies $\mathcal{D}_K$ ($\mathcal{D}_K = 0$ for some values of K, see Ref. [14]). These gluon KK modes are five-dimensional fields whose effects at energies below their masses are approximately described by four-quark operators.

At scales between π/L_z and the string scale, M_s, the dynamics includes both light gluon KK modes and four-quark operators induced by the heavier gluon KK modes. Although each gluon KK mode is weakly coupled, the number of gluon KK modes may be sufficiently large [6] such that the loop expansion breaks down. In order to analyze the effects of this nonperturbative theory below some scale Λ, we approximate the dynamics of

the gluons with momentum in the z dimensions by a five-dimensional effective theory with four-quark operators. The matching between the five-dimensional low-energy theory and the $(4+\delta)$-dimensional theory is likely to require the scale Λ of the four-quark operators to be somewhere between π/L_z and M_s.

By imposing that the loop expansion parameter [14] becomes of order one at M_s, we can estimate the separation between π/L_z and M_s. For $\delta \gtrsim 3$, the density of KK modes is large and it turns out that M_s is only about twice π/L_z. Therefore, the uncertainty in Λ is not worrisome.

The relevant piece of the five-dimensional Lagrangian density, involving the four-dimensional $\psi_L(x^\mu)$ field and the five-dimensional $\chi(x^\mu, y)$ and massless gluon fields is given at the scale Λ by

$$\mathcal{L}_5(x^\mu, y) = \delta(y-y_0) i\overline{\psi}_L \gamma^\mu D_\mu \psi_L + \overline{\chi}\left(i\gamma^\mu D_\mu - \gamma_5 D_y\right)\chi - \frac{1}{2g_5^2}\mathrm{Tr}(F^{ab}F_{ab}) + \mathcal{L}_{\mathrm{CS}}(G) + \mathcal{L}_{\mathrm{int}} \ . \tag{3.1}$$

F^{ab} is the gluon field strength, $\mathcal{L}_{\mathrm{CS}}(G)$ is the Chern-Simons term for the gluon field [see eqs. (2.8) and (2.11)], and D is the covariant derivative:

$$\begin{aligned} D_\mu &= \partial_\mu - G_\mu \ , \\ D_y &= \frac{\partial}{\partial y} - G_y \ , \end{aligned} \tag{3.2}$$

with $G_{\mu,y} = -ig_5 G^r_{\mu,y} T^r$ being five-dimensional gluon fields (the zero modes from the KK expansion in the z directions) polarized in the x^μ and y directions, respectively. The five-dimensional strong coupling constant, g_5, has dimension $(\mathrm{mass})^{-1/2}$.

The $\mathcal{L}_{\mathrm{int}}$ part of the $\mathcal{L}_5$ Lagrangian includes the four-quark operators induced by gluon KK mode exchange. Although the $SU(3)_C$ interactions are flavor universal, the four-quark operators contained in $\mathcal{L}_{\mathrm{int}}$ are not, because different quark fields are assumed to be localized at different positions in the extra dimensions. For example, all $SU(2)_W$ singlet quark fields other than t_R and its excitations may be localized at $z = z_R > 0$, and the $SU(2)_W$ doublet quarks of the first two generations may be localized at $z = z_L > 0$ with $z_L \neq z_R$. In this case the terms from $\mathcal{L}_{\mathrm{int}}$ that could lead to large quark masses in the low energy theory, namely the left-right current-current terms, are exponentially suppressed unless they involve only ψ_L and χ.

The four-quark operators involving $\psi_L(x^\mu)$ and $\chi(x^\mu, y)$, obtained by integrating out

the five-dimensional gluon KK excitations, are given by

$$\mathcal{L}_{\rm int}(x,y) = -\frac{cg_5^2}{2\Lambda^2}\left\{\left[\delta(y-y_0)\left(\overline{\psi}_L\gamma^\mu T^r\psi_L\right) + \left(\overline{\chi}\gamma^\mu T^r\chi\right)\right]^2 + \left(\overline{\chi}\gamma_5 T^r\chi\right)^2\right\} \ , \qquad (3.3)$$

where $c \gg 1$ is a dimensionless coefficient obtained by summing over the effects of the gluon KK modes, and T^r are $SU(3)_C$ generators.

These four-quark operators may be Fierz transformed, with the result

$$\mathcal{L}_{\rm int}(x,y) = \frac{cg_5^2}{\Lambda^2}\left\{\delta(y-y_0)\left(\overline{\psi}_L\chi\right)\left(\overline{\chi}\psi_L\right) + \frac{5}{16}\left[\left(\overline{\chi}\chi\right)^2 - \frac{1}{3}\left(\overline{\chi}\gamma_5\chi\right)^2\right]\right\} + \ldots \ , \qquad (3.4)$$

where the ellipsis stand for vectorial and tensorial four-quark operators, which are irrelevant at low energies.

3.2 The five-dimensional effective potential

The operators shown in $\mathcal{L}_{\rm int}$ provide attractive interactions which give rise to bound states: a four-dimensional weak-doublet complex scalar, $H(x^\mu)$, and a five-dimensional gauge singlet real scalar, $\varphi(x^\mu, y)$. These composite scalars are propagating degrees of freedom only below the compositeness scale. According to our approximation in which the full KK mode dynamics is described at low energy by a five-dimensional theory with four-quark operators, the compositeness scale is identified with Λ.

At the compositeness scale the composite scalars are non-propagating, and the four-quark operators may be replaced by Yukawa interactions between the scalars and their constituents. The first two terms shown in (3.4) are equivalent with

$$\mathcal{L}_c[\Lambda] = -\delta(y-y_0)\left[\sqrt{cg_5^2}\left(\overline{\chi}\psi_L\right)H + \Lambda^2 H^\dagger H\right] - \sqrt{\frac{5}{8}cg_5^2}\left(\overline{\chi}\chi\right)\varphi - \frac{\Lambda^2}{2}\varphi^2 \ , \qquad (3.5)$$

as can be seen by integrating out $H(x^\mu)$ and $\varphi(x^\mu, y)$. The last term in eq. (3.4) gives rise to a five-dimensional pseudo-scalar. However, the coefficient of this term is suppressed by the factor of 1/3, such that the pseudo-scalar is not sufficiently deeply-bound to be relevant at energies below the compositeness scale.

At scales $\mu < \Lambda$, the Yukawa interactions induce kinetic terms for the scalars:

$$\begin{aligned}\mathcal{L}_c[\mu] \ &= \ \delta(y-y_0)\left[Z_H(\mu)D^\nu H^\dagger D_\nu H - \sqrt{cg_5^2}\left(\overline{\psi}_L\chi\right)H\right] \\ &\quad + Z_\varphi(\mu)\partial^a\varphi\partial_a\varphi - \sqrt{\frac{5}{8}cg_5^2}\left(\overline{\chi}\chi\right)\varphi - V(\mu) \ . \end{aligned} \qquad (3.6)$$

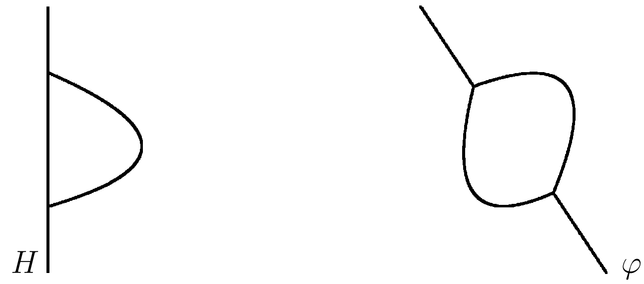

Figure 2: Large-N_c contributions to the composite scalar self-energies. The vertical lines are four dimensional fields localized at $y = y_0$, and the curved or slanted lines are five-dimensional fields. The external lines represent the H and φ, while in the loops run the ψ_L and χ quarks.

The wave function renormalization Z_H can be determined by computing the self-energy of the weak-doublet in the large-N_c limit (see Fig. 2):

$$Z_H(\mu) = 2N_c \frac{cg_5^2}{L} \sum_{j\geq 0} \frac{\cos^2(\pi j y_0/L)}{(1+\delta_{j0})} \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k^\mu k_\mu \left[k^\nu k_\nu - (\pi j/L)^2\right]} \ . \tag{3.7}$$

The integral is logarithmic divergent, and has to be cut-off at Λ. The sum over the momenta in the fifth dimension is also divergent, and is cut-off at $n_{\rm KK} \approx \Lambda L/\pi$. The integral has also an infrared cut-off at μ.

The wave function renormalization for the φ scalar has a more complicated form, due to the two χ propagators involved [see eq. (2.6)]. Keeping only the leading divergent piece, we find

$$Z_\varphi(\mu) \approx \frac{5}{4} N_c \frac{cg_5^2}{L} \sum_{j\geq 0} \int \frac{d^4k}{(2\pi)^4} \frac{-i}{\left[k^\nu k_\nu - (\pi j/L)^2\right]^2} \ . \tag{3.8}$$

Note that the wave function renormalization for $\partial\varphi/\partial y$ is somewhat arbitrary (it can be absorbed in the mass term for φ), and does not have to be the same as $Z_\varphi(\mu)$. In $\mathcal{L}_c[\mu]$ we have chosen these two wave function renormalizations to be the same for convenience.

The scalar potential includes mass and quartic terms,

$$V(\mu) = \delta(y-y_0)\left[\frac{\tilde{\lambda}_H}{2}\left(H^\dagger H\right)^2 + \frac{\tilde{\lambda}_0 L}{2} H^\dagger H \varphi^2 + \tilde{M}_H^2 H^\dagger H\right] + \frac{\tilde{\lambda}_\varphi L}{4!}\varphi^4 + \frac{\tilde{M}_\varphi^2}{2}\varphi^2 \ , \tag{3.9}$$

as well as higher-dimensional terms which we will ignore. The mass parameters computed in the large-N_c limit are given by

$$\begin{aligned} \tilde{M}_H^2(\mu) &= \Lambda^2 - 4N_c \frac{cg_5^2}{L} \sum_{j\geq 0} \frac{\cos^2(\pi j y_0/L)}{1+\delta_{j0}} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^\nu k_\nu - (\pi j/L)^2} \ , \\ \tilde{M}_\varphi^2(\mu) &\approx \Lambda^2 - \frac{5}{2} N_c \frac{cg_5^2}{L} \sum_{j\geq 0} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^\nu k_\nu - (\pi j/L)^2} \ . \end{aligned} \tag{3.10}$$

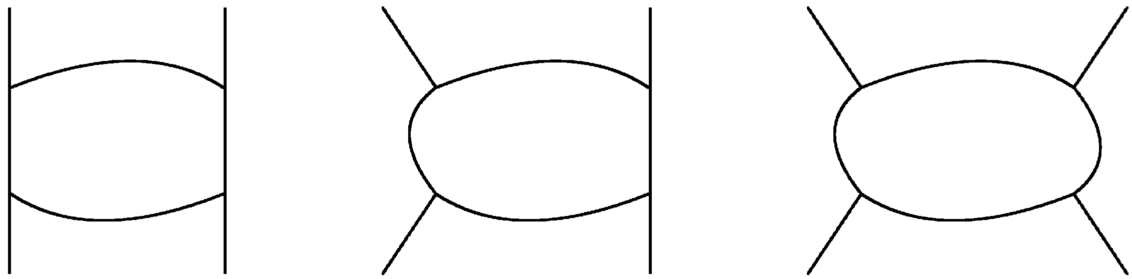

Figure 3: Large-N_c contributions to the $\tilde{\lambda}_H, \tilde{\lambda}_0$ and $\tilde{\lambda}_\varphi$ quartic couplings. The lines represent fields as explained in the caption of Fig. 2.

In the expression for $\tilde{M}_\varphi^2$ we have kept again only the leading divergent piece.

In the large-N_c limit, the leading contribution to the dimensionless quartic coupling, $\tilde{\lambda}_H$, is given by a quark loop with alternating χ and ψ_L propagators (Fig. 3). Therefore, the result can be written as a double sum over the χ momenta in the fifth dimension:

$$\tilde{\lambda}_H(\mu) = 8N_c \left(\frac{cg_5^2}{L}\right)^2 \sum_{j_{1,2}\geq 0} f_{j_1 j_2} \cos^2\left(\frac{\pi j_1 y_0}{L}\right) \cos^2\left(\frac{\pi j_2 y_0}{L}\right) \tag{3.11}$$

where we have defined

$$f_{j_1 j_2} \equiv \frac{1}{(1+\delta_{j_1 0})\,(1+\delta_{j_2 0})} \int \frac{d^4k}{(2\pi)^4} \frac{-i}{[k^\nu k_\nu - (\pi j_1/L)^2]\,[k^\rho k_\rho - (\pi j_2/L)^2]} \,. \tag{3.12}$$

The coefficients of the quartic terms involving φ have mass dimension -1. The factors of L are introduced in eq. (3.9) such that the $\tilde{\lambda}_0$ and $\tilde{\lambda}_\varphi$ quartic couplings are dimensionless:

$$\begin{aligned} \tilde{\lambda}_0(\mu) &\approx \frac{5}{4} N_c \left(\frac{cg_5^2}{L}\right)^2 \sum_{j_{1,2,3}\geq 0} f_{j_1 j_3} \frac{d_{j_1 j_2} d_{j_3 j_2}}{(1+\delta_{j_2 0})} \cos\left(\frac{\pi j_1 y_0}{L}\right) \cos\left(\frac{\pi j_3 y_0}{L}\right) \\ \tilde{\lambda}_\varphi(\mu) &\approx \frac{75}{128} N_c \left(\frac{cg_5^2}{L}\right)^2 \sum_{j_{1,2,3,4}\geq 0} f_{j_1 j_2} \frac{d_{j_1 j_2} d_{j_3 j_2} d_{j_3 j_4} d_{j_1 j_4}}{(1+\delta_{j_3 0})\,(1+\delta_{j_4 0})} \,. \end{aligned} \tag{3.13}$$

When all the φ fields from the φ^4 interaction have momentum π/L in the y direction, we obtain:

$$d_{j_1 j_2} \equiv \delta_{j_2, j_1+1} - \delta_{j_2, j_1-1} + \delta_{j_2, 1-j_1} \,. \tag{3.14}$$

To evaluate all these parameters, we assume that the number of the KK modes in the y direction, $n_{\rm KK}$, is large enough so that we can approximate the sums over KK states by integrals. The expressions obtained for the parameters are given in the Appendix.

The kinetic terms in $\mathcal{L}_c[\mu]$ [see eq. (3.6)] may be cannonically normalized by redefining the scalar fields: $H \to H\sqrt{Z_H}$ and $\varphi \to \varphi\sqrt{Z_\varphi}$. In this case, the terms in the scalar potential have the same form as in eq. (3.9), but with appropriately normalized coefficients. We denote the new parameters by dropping the tilde from the corresponding symbols used in eq. (3.9). The squared-masses are given by

$$\begin{aligned} M_H^2 &= \frac{\tilde{M}_H^2}{Z_H} \approx \frac{2\Lambda^2}{F_1(y_0)}\left[\frac{4\pi^2}{n_{\rm KK} N_c c g_s^2} - F_3(y_0)\right] \\ M_\varphi^2 &= \frac{\tilde{M}_\varphi^2}{Z_\varphi} \approx \frac{2\Lambda^2}{F_2}\left(\frac{32\pi^2}{5 n_{\rm KK} N_c c g_s^2} - F_4\right) \end{aligned} \tag{3.15}$$

We have used here the four-dimensional $SU(3)_C$ gauge coupling, g_s, obtained in terms of the five-dimensional coupling by integrating over the y dimension:

$$g_s = \frac{g_5}{\sqrt{L}} \ . \tag{3.16}$$

The dependence of M_H^2 on the position y_0 of the ψ_L doublet is encoded in the $F_{1,3}(y_0)$ functions, which are symmetrical under the $y_0 \to L - y_0$ reflection. $F_1(y_0)$ and $F_3(y_0)$ have values of order one, with maxima on the boundary and minima at $y = L/2$. F_2 and F_4 are constant functions on the $[0, L]$ interval because the φ mass is induced by interactions which conserve momentum in the y dimension. These functions are given in terms of divergent sums and integrals and depend on the cut-off procedure. In the Appendix we estimate them in the continuum limit with a specific cut-off.

Similarly, the quartic couplings may be written as follows:

$$\begin{aligned} \lambda_H &= \frac{\tilde{\lambda}_H}{Z_H^2} \approx \frac{32\pi^2 F_5(y_0)}{N_c\,[F_1(y_0)]^2} \ , \\ \lambda_0 &= \frac{\tilde{\lambda}_\varphi}{3 Z_H Z_\varphi} \approx \lambda_\varphi \frac{F_6(y_0)}{F_1(y_0)} \ , \\ \lambda_\varphi &= \frac{\tilde{\lambda}_\varphi}{Z_\varphi^2} \approx \frac{16\pi^2}{n_{\rm KK} N_c F_2} \ . \end{aligned} \tag{3.17}$$

Like the other F-functions written in the Appendix, $F_{5,6}(y_0) \sim 1$. Note that λ_H is enhanced by an $n_{\rm KK}$ factor compared with the other quartic couplings.

4 Four-dimensional phenomenology

The squared-mass parameters from the five-dimensional potential may turn negative if the four-quark operators induced by gluon KK modes are strong enough. Therefore, the four-dimensional field, H, and the five-dimensional real scalar, φ, may acquire VEVs. The composite weak-doublet H may be identified with the Standard Model Higgs doublet. In this Section we discuss the scalar spectrum and its phenomenological implications, and we estimate the top quark mass.

4.1 Higgs boson mass

First, we consider the case in which ψ_L is located at the boundary ($y_0 = 0$). An inspection of the squared-masses computed in the large-N_c limit [see eq. (3.15)] reveals that only M_H^2 should become negative because the coupling in the $\overline{\chi}\psi H$ channel is stronger than the coupling in the $\overline{\chi}\chi\varphi$ channel. In addition, the four-dimensional quartic coupling involving both H and φ vanishes in this case because the φ has a zero wave function on the boundary. This implies that there is no mixing between H and φ. Therefore, the φ has no effect on the Higgs potential in this case. The H acquires a VEV while the KK modes of φ have masses above the compactification scale.

The effective theory below the compactification scale is given by the Standard Model. The compositeness of the Higgs doublet is not manifest at low-energy. However, as a remnant of the strong dynamics that binds the Higgs, the quartic Higgs coupling is large, $\lambda_H \gg 1$. The Higgs boson mass M_{h^0}, given at tree level by $v\sqrt{\lambda_H(v)}$, appears to be above 1 TeV. The tree level estimate, though, should not be taken too seriously due to the large λ_H. Because the theory that gives rise to the composite Higgs boson is unitary (above the M_s scale, the unitarity should be enforced by quantum gravitational effects), the Higgs mass is below the bound imposed by the unitarity of the WW scattering cross section in the Standard Model. Once the non-perturbative corrections to M_{h^0} are included, we expect $M_{h^0} \sim \mathcal{O}(1/2)$ TeV. Generically, when the ψ_L is at $y_0 = 0$, the Higgs boson is a broad resonance.

Note that such a heavy Higgs boson is perfectly compatible with the electroweak precision data. The often quoted upper bound on the Higgs boson based on the fit to the electroweak data is valid only if there are no fields or interactions beyond the Standard Model [22]. In our case, however, there are KK excitations of the Standard Model gauge bosons and t_R, with masses in the TeV range. In their presence, a heavier Higgs boson is

not only allowed, but potentially preferred by the fit to the data. This has been shown in the context of extra dimensions in ref. [23]. Specifically, the shift in the electroweak observables due to the mixing of the W and Z with their KK excitations reproduces that due to a light Higgs boson (when the Higgs is trapped on a 3+1-dimensional wall, like in our case). Furthermore, when a vector-like quark identical with our KK modes of t_R is added to the Standard Model, the fit to the electroweak data yields a heavy Higgs for a vector-like quark mass around 5 TeV [4]. Of course, when the vector-like quark is much heavier, or equivalently the compactification scale in our model is increased, one recovers the Standard Model in the decoupling limit. Therefore, the $y_0 = 0$ case is consistent with the electroweak precision data only if the compactification scale is not above $\mathcal{O}(10 \text{ TeV})$.

In the other case, where the ψ_L fermion doublet is located in the middle of the interval occupied by χ, *i.e.* $y_0 \sim L/2$, both H and φ may develop VEVs. (Note that eqs. (3.15) imply that for $F_3(y_0) \approx 5F_4/8$ both M_H^2 and M_φ^2 turn negative at some particular value of $n_{\rm KK}\, c g_s^2$.) Since the Higgs VEV is below the compactification scale, it is appropriate to integrate first over the fifth dimension, and only afterwards to minimize the potential. The five-dimensional real scalar decomposes in a tower of KK modes

$$\varphi(x^\mu, y) = \sqrt{\frac{2}{L}} \sum_{j\geq 1} \varphi_j(x^\mu) \sin\left(\frac{\pi j y}{L}\right) \ . \tag{4.1}$$

It is likely that only one or the first few modes of φ are light enough to have a significant mixing with the H.

For simplicity, we consider that the Higgs field mixes with one φ mode. The four-dimensional potential may be obtained readily from eq. (3.9):

$$V_4 = \frac{\lambda_H}{2}\left(H^\dagger H\right)^2 + \lambda_0(y_0)\sin^2\!\left(\frac{\pi y_0}{L}\right) H^\dagger H \varphi_1^2 + \frac{\lambda_\varphi}{16}\varphi_1^4 + M_H^2 H^\dagger H + \frac{1}{2}\left(M_\varphi^2 + \frac{\pi^2}{L^2}\right)\varphi_1^2 \, . \tag{4.2}$$

After the scalar potential is minimized and the scalar fields shifted, we find the following mass terms for the two light neutral degrees of freedom:

$$\frac{1}{2}\,(h,\phi)\begin{pmatrix} \lambda_H v^2 & 2\lambda_0(y_0) v u \sin^2\!\left(\frac{\pi y_0}{L}\right) \\ 2\lambda_0(y_0) v u \sin^2\!\left(\frac{\pi y_0}{L}\right) & \frac{1}{2}\lambda_\varphi u^2 \end{pmatrix}\begin{pmatrix} h \\ \phi \end{pmatrix} , \tag{4.3}$$

where $v \approx 246$ GeV and u is the φ_1 VEV.

In the limit $\lambda_H v^2 \ll \lambda_\varphi u^2$, the mixing of h and ϕ decreases the Higgs boson mass:

$$M_{h^0}^2 \approx \lambda_H v^2 \left[1 - \frac{8[\lambda_0(y_0)]^2}{\lambda_H \lambda_\varphi} \sin^4\!\left(\frac{\pi y_0}{L}\right)\right] \tag{4.4}$$

For $y_0 = L/2$, the M_{h^0} decreases by $\sim (70/n_{\rm KK})\%$. This value is derived using the λ_H given in eq. (3.17). As argued before, we expect that the quantum corrections actually drive λ_H smaller, which would lead to an enhancement of the change in M_{h^0} due to mixing. If more φ modes participate in the mixing, the decrease in M_{h^0} becomes even more significant. Perhaps the Higgs boson may be driven close to the current LEP limit. Unfortunately, it is hard to study the scalar spectrum in general, with all KK modes included, especially given that the parameters of the full effective potential are not accurately known.

In the other limit, where $\lambda_H v^2 \gg \lambda_\varphi u^2$, the $h-\phi$ mixing may be ignored. The Higgs boson remains heavy, but the physical ϕ^0 scalar could be very light. Its mass,

$$M_\phi^0 \approx u\sqrt{\frac{\lambda_\varphi}{2}} \ , \tag{4.5}$$

is not constrained by the consistency of the model. The experimental lower bounds on a neutral scalar which couples only to the top quark and the Higgs boson are quite weak [5]. It is therefore possible that the Higgs boson decays into ϕ^0 pairs, giving rise to unusual signals at future collider experiments [24].

4.2 Top quark mass prediction

We can now predict the top-quark mass as a function of the number of KK modes and the position y_0 of the ψ_L doublet. The fermion couplings to the composite scalars are given by eq. (3.6). Upon normalization of the scalar kinetic terms and integration over the y dimension, the Yukawa couplings become:

$$\begin{aligned} &-\xi_t \sum_{j=1}^{n_{\rm KK}} \left(\frac{2}{1+\delta_{j0}}\right)^{1/2} \cos\left(\frac{\pi j y_0}{L}\right) \overline{\chi}_R^j \psi_L H \\ &-\xi_\chi \sum_{j_{1,2,3}=1}^{n_{\rm KK}} \left(\delta_{j_3,j_1+j_2} - \delta_{j_3,j_1-j_2} + \delta_{j_3,j_2-j_1}\right) \overline{\chi}_L^{j_1} \chi_R^{j_2} \varphi_{j_3} + \text{h.c.} \end{aligned} \tag{4.6}$$

Note that the Yukawa couplings of the Higgs doublet depend on the position in the fifth dimension. The zero-mode of χ, namely t_R, has a Yukawa coupling to H given by

$$\xi_t = \frac{2\sqrt{2}\pi}{\sqrt{N_c n_{\rm KK} F_1(y_0)}} \ . \tag{4.7}$$

The Yukawa couplings of the φ KK modes are position-independent due to momentum conservation at the $\overline{\chi}\chi\phi$ vertex:

$$\xi_\chi = \frac{2\pi}{\sqrt{N_c n_{\rm KK} F_2}} \ . \tag{4.8}$$

The fermion masses for the t_L component of ψ_L and the KK modes of χ form a $(n_{\rm KK}+1)\times(n_{\rm KK}+1)$ matrix. There are two contributions to the elements of this mass matrix. First, the Yukawa interactions give contributions determined by replacing the H and φ^j scalars with their VEVs in eq. (4.6). Second, the kinetic term of the five-dimensional χ field yields the usual KK mass terms:

$$\sum_{j=1}^{n_{\rm KK}} \frac{\pi j}{L} \overline{\chi}_L^j \chi_R^j \ . \tag{4.9}$$

In the case where $y_0 = 0$, the fermion mass matrix is easy to write:

$$\left(\overline{t}_L\,,\ \overline{\chi}_L^1\,,\ \overline{\chi}_L^2\,,\ \ldots\right)\begin{pmatrix} \frac{\xi_t v}{\sqrt{2}} & \xi_t v & \xi_t v & \ldots \\ 0 & \frac{\pi}{L} & 0 & \ldots \\ 0 & 0 & \frac{2\pi}{L} & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{pmatrix}\begin{pmatrix} t_R \\ \chi_R^1 \\ \chi_R^2, \\ \ldots \end{pmatrix} + \text{h.c.} \tag{4.10}$$

The top-quark mass (in the limit where we ignore the small mixing of the top with the charm and up) is given by the lowest eigenvalue of the above mass matrix. It is amusing that this matrix has the same form as the one for neutrino masses given in ref. [25]. Note that our assumption that the KK-gluons in the z dimensions may be integrated out below the cut-off scale Λ (see Section 3) is legitimate provided $\Lambda \gg \pi/L$. Thus, to be consistent we must impose $n_{\rm KK} \gtrsim 10$. Expanding in $(vL/\pi)^2 \ll 1$, and taking $n_{\rm KK} \gg 1$, we find

$$m_t \approx \frac{\xi_t v}{\sqrt{2}}\left[1 - \frac{3}{2}\left(\xi_t v L\right)^2\right] \ . \tag{4.11}$$

For $L \lesssim 1$ TeV^{-1}, the second term gives a small correction ($\lesssim 1/n_{\rm KK}$) to m_t. Therefore, the top mass is predicted in terms of $n_{\rm KK}$:

$$m_t \approx \frac{600 \text{ GeV}}{\sqrt{n_{\rm KK}}} \ . \tag{4.12}$$

The measured top mass can be used to determine the number of top KK modes:

$$n_{\rm KK} \approx 12. \tag{4.13}$$

The number of top KK modes is related to the cut-off scale $\Lambda \approx n_{\rm KK}\pi/L$, which is of the order of the string scale M_s. If the first KK modes have a mass of a few TeV, then the above prediction determines the scale of quantum gravity $M_s \sim 30$ TeV.

Furthermore, given that a cut-off scale significantly above ~ 50 TeV would require excessive fine-tuning (we assume that the theory is not supersymmetric below the string scale), we find a naturalness upper bound $n_{\rm KK} \lesssim 20$. Therefore, instead of using the measured top mass to determine the number of KK modes, we may reverse the argument and determine the typical values of the top mass in our model. For $10 \lesssim n_{\rm KK} \lesssim 20$, we find a range, 130 GeV $\lesssim m_t \lesssim$ 190 GeV, which within the theoretical uncertainties is in agreement with the measured value.

When the ψ_L is placed in the middle of the thick brane occupied by χ, *i.e.* $y_0 \sim L/2$, some of the φ KK modes may acquire VEVs, as discussed in section 4.1. Therefore, the fermion mass matrix becomes more complicated to analyze. If only the first φ KK mode has a non-zero VEV, u, and $u \ll \pi/L$, then the top mass may be computed as in the $y_0 = 0$ case. The only notable difference is that m_t is enhanced by a factor of $\sqrt{F_1(0)/F_1(y_0)}$. This factor reaches its maximum of $\sqrt{2}$ at $y_0 = L/2$. It appears that the upper end of the interval for $n_{\rm KK}$ is preferred in this case.

In the more general case, where the VEVs of some φ_j are comparable with the compactification scale, one could imagine that the preferred value of the string scale is lower, $M_s \sim 10$ TeV. In such a situation our estimates would no longer be reliable, but the qualitative picture of a composite Higgs doublet bound out of ψ_L and a tower of t_R KK modes might remain valid.

Finally, we emphasize that the masses of the light quark and leptons may easily be accommodated in our scenario. For example, certain four-quark operators presumed to be generated at the string scale with coefficients of order one in M_s units, give rise in the low-energy effective theory to the Standard Model Yukawa couplings [5].

5 Conclusions

Electroweak symmetry breaking remains the foremost problem facing elementary particle physics at this moment. We expect to come to understand it in scientific detail in the next decade with the Tevatron and the LHC.

We find it remarkable that the ingredients needed for a dynamical explanation of the origin of the electroweak scale, which we often have previously invoked in model building attempts (e.g., topcolor, vector-like fermions, strong coupling Nambu–Jona-Lasinio dynamics, etc.), are seemingly presented automatically in theories with extra-dimensions at the $\sim$ TeV scale.

In this paper we have explicitly constructed a "demo–model" of the dynamics in which the only fundamental fields below the string scale are the $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge bosons and the three generations of quarks and leptons, living in a higher-dimensional compact space.

Strong dynamics comes from the existing QCD gauge group, which has a large coupling strength above the compactification scale, due to the large number of KK-modes. The KK-mode gluons act like degenerate octets of colorons which, via exchange, give rise to four-fermion operators. Thus follows an NJL approximation to the dynamics induced by these operators.

We find that various attractive channels lead to the formation of scalar bound-states. The Higgs doublet channel corresponds to $\bar{\chi}\psi_L$ where χ is the right-handed top quark field which we take to live in the bulk. While χ has a chiral zero-mode by construction, which is the t_R, the Higgs doublet emerges as a bound-state involving a linear combination of the active KK-modes inherent in χ. In the effective theory the large number of active KK-modes, $n_{\rm KK}$, controls the dynamics, and naturally leads to a tachyonic mass term for the Higgs at low energies, and thus electroweak symmetry breaking. We also expect various gauge-singlet composite bosons to form in channels such as $\bar{\chi}\chi$, which somewhat complicate the discussion of the low energy spectroscopy. A low mass Higgs boson may emerge through mixing between the primary composite Higgs and the extra composite singlets.

Our model is largely intended to illustrate what can happen in the extra-dimensional theories. It is hardly unique. The only selection criterion seems to be the assignment of Standard Model fields to the world-brane or into the bulk, in various dimensional configurations. We believe that, once the brane/bulk field assignments are made in this manner, much of the dynamics we describe is forced to happen. New strong dynamics is therefore natural and expected to occur in these theories. The experimental confirmation of a strongly interacting Higgs sector beyond the Standard Model would, though not "imply", nonetheless lend support to the notion of extra dimensions at the TeV scale.

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Appendix: effective potential parameters

In this Appendix we give the formulae for the parameters of the low-energy effective Lagrangian in the continuous approximation by replacing sums of the KK states with the momentum integrals in the fifth direction.

Cutting off the integrals at Λ and replacing $\Lambda L/\pi$ by $n_{\rm KK}$, we find the following wave function renormalizations at low-energy ($\sim L^{-1}$)

$$\begin{aligned} Z_H &\approx n_{\rm KK}\frac{N_c c g_5^2}{16\pi^2 L}\, 2\, F_1(y_0)\ , \\ Z_\varphi &\approx n_{\rm KK}\frac{N_c c g_5^2}{16\pi^2 L}\, \frac{5}{4}\, F_2\ . \end{aligned} \tag{A.1}$$

Likewise, we find the parameters from the five-dimensional effective potential (see section 3.2):

$$\begin{aligned} \tilde{M}_H^2 &\approx \Lambda^2\left[1 - n_{\rm KK}\frac{N_c c g_5^2}{16\pi^2 L}\, 4\, F_3(y_0)\right]\ , \\ \tilde{M}_\varphi^2 &\approx \Lambda^2\left[1 - n_{\rm KK}\frac{N_c c g_5^2}{16\pi^2 L}\, \frac{5}{2}\, F_4\right]\ , \\ \tilde{\lambda}_H &\approx n_{\rm KK}^2\frac{N_c}{16\pi^2}\left(\frac{c g_5^2}{L}\right)^2\, 8\, F_5(y_0)\ , \\ \tilde{\lambda}_0 &\approx n_{\rm KK}\frac{N_c}{16\pi^2}\left(\frac{c g_5^2}{L}\right)^2\, 5\, F_6(y_0)\ , \\ \tilde{\lambda}_\varphi &\approx n_{\rm KK}\frac{N_c}{16\pi^2}\left(\frac{c g_5^2}{L}\right)^2\, \frac{75}{8}\, F_2\ , \end{aligned} \tag{A.2}$$

where the F-functions are defined by

$$\begin{aligned} F_1(y_0) &= \int_0^1 p^2 dp^2 \int_0^1 dq \cos^2(q\Lambda y_0)\frac{1}{p^2(p^2+q^2)} \\ F_2 &= \int_0^1 p^2 dp^2 \int_0^1 dq \frac{1}{(p^2+q^2)^2} \\ F_3(y_0) &= \int_0^1 p^2 dp^2 \int_0^1 dq \cos^2(q\Lambda y_0)\frac{1}{p^2+q^2} \end{aligned}$$

$$\begin{aligned}
F_4 &= \int_0^1 p^2 dp^2 \int_0^1 dq \frac{1}{p^2+q^2} \\
F_5(y_0) &= \int_0^1 p^2 dp^2 \int_0^1 dq \cos^2(q\Lambda y_0) \int_0^1 dq' \cos^2(q'\Lambda y_0) \frac{1}{(p^2+q^2)(p^2+q'^2)} \\
F_6(y_0) &= \int_0^1 p^2 dp^2 \int_0^1 dq \cos^2(q\Lambda y_0) \frac{1}{(p^2+q^2)^2} .
\end{aligned} \tag{A.3}$$

For ψ_L localized at the boundary, we have

$$\begin{aligned}
F_1(0) &= \frac{\pi}{2} + \ln 2 \approx 2.26 , \\
F_6(0) &= F_2 = \frac{\pi}{4} + \ln 2 \approx 1.48 , \\
F_3(0) &= F_4 = \frac{1}{3} + \frac{\pi}{6} - \frac{1}{3}\ln 2 \approx 0.63 , \\
F_5(0) &\approx 2.71 .
\end{aligned} \tag{A.4}$$

If ψ_L is localized in the middle of the $[0, L]$ interval and $\Lambda y_0 \gg 1$, the $\cos^2(q\Lambda y_0)$ weigth factor averages to $1/2$, and therefore

$$\begin{aligned}
F_{1,3,6}\left(y_0 \sim L/2\right) &\approx \frac{1}{2} F_{1,3,6}(0) , \\
F_5\left(y_0 \sim L/2\right) &\approx \frac{1}{4} F_5(0) .
\end{aligned} \tag{A.5}$$

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